Thrust Augmenting Ejectors, Part I

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An analysis of the flow of compressible fluids through a thrust augmenting ejector is presented. The mixing process has been analyzed under the assumption that the process is carried to completion in a duct of constant cross section. Two distinctly different flows after complete mixing have been described. The first solution represents a subsonic mixed flow and the second solution a supersonic mixed flow. These two solutions are related in the same manner as the flows on each side of a normal shock wave. The existence of flow patterns for optimal thrust augmentation and corresponding ejector geometries for each solution, and a flow pattern and ejector geometry corresponding to the limit imposed by the second law of thermodynamics for the second solution have been described. This Part I document presents maps illustrating the ideal, optimal performance of ejectors designed under the first solution with a mixing duct which is 25 times that of the minimum area of the primary jet, for flight Mach numbers from 0 to 2, over a wide range of primary jet stagnation pressures and temperatures. The influence of major losses on the performance has been illustrated by example.

Nomenclature

a	= primary jet area
a_*	= primary jet throat area or area at ambient pressure exhaust
M	= Mach number
$\dot{m}$	= mass flow rate
P_0	= stagnation pressure
P	= pressure
R	= gas constant
r	= entrainment or mass flow ratio, $=\dot{m}_i/\dot{m}_p$
T	= temperature
T_0	= stagnation temperature
U	= secondary or mixed flow velocity
V	= primary or injected flow velocity
X	= duct width or area
α	= area ratio, $=X_2/a_1$
α_*	= area ratio, $=X_2/a_*$
γ	= ratio of specific heats (C_p/C_v), $=1.4$ for data presented
ΔP	= primary jet pressure rise, $=P_{0p}-P_{0\infty}$
ΔS	= total entropy production due to mixing
ΔT	= primary jet temperature rise, $=T_{0p}-T_{0\infty}$
ρ	= mass density
ϕ	= thrust augmentation

Subscripts

c	= choking
i	= induced or secondary flow
p	= primary flow
$1,2,3$	= ejector stations
∞	= ambient or freestream conditions

Introduction

HISTORICALLY, several different types of devices have been given the generic name "ejector" simply because they relied upon the induction of a quantity of ambient fluid into a duct by some form of interaction with a stream of energized fluid in the duct. Early work in the analysis of this type of flow (e.g., Refs. 1 and 2) was performed in order to determine the feasibility of such devices for pumping an ambient or secondary fluid over a pressure rise. The secondary fluid was pumped from a lower to a higher pressure as a result of the interaction with the energized (primary) jet stream.

The generic name ejector has also been applied to similarly appearing devices intended to recover the portion of the isentropic thrust force which cannot be realized when exhausting a supercritical pressure ratio flow through a nozzle that is only convergent. These devices, commonly referred to as blow-in-doors (BID's) were treated analytically,[3,4] mainly by methods utilizing inviscid flow interactions and requiring a priori definition of their geometry; thus, performance estimates were limited to some simple geometries conceived by various investigators.

Thrust augmenting ejectors, as treated in this paper and in Ref. 5, are quite different devices, primarily due to their operating regime, their intended function, and corresponding performance criterion. For example, the thrust augmenting ejector operates at an overall secondary flow pressure ratio of 1 and its performance is measured by the ratio of its momentum increment to that of its reference jet; conditions which are of no interest to the jet pump. Further, thrust augmenting ejectors are intended to achieve increments of thrust far in excess of the thrust of their primary jets when expanded to ambient pressure; a goal far in excess of BID's. The achievement of this goal depends upon the achievement of efficient ingestion of ambient fluid, rapid thermal and kinetic mixing with the energized fluid, and efficient discharge of the mixture to ambient pressure. The analysis presented provides means for determination of the local flow properties, the performance, and the general geometrical requirements of the ejector duct.

This work was intended to provide a basis for the determination of the potential advantages of ejector thrust augmentation during translational motion and to evaluate the influence of the major internal ejector losses on performance. Other losses associated with the integration of the ejector and the aircraft could have substantial influence on the system performance, but since these losses are design dependent, their effects are not evaluated in this general study.

Analysis

Thrust augmenting ejectors utilize a process in which a primary, energized fluid stream is mixed with a quantity of ambient fluid in a duct configured to achieve a momentum increment of the mixed flow which is large compared to that of the primary stream when exhausted to ambient pressure.

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The analysis presented here utilizes several simplifying assumptions but attempts to retain those characteristics which are essential for an evaluation of the optimal performance and the corresponding ejector geometry. To accomplish this it is essential to develop the flow characteristics within the ejector in a generalized mathematical form, rather than utilizing methods which require a priori selection of the geometry. The major assumptions utilized in this analysis are as follows.

1) All fluids are compressible and thermally and calorically perfect.

2) Skin friction and blockage losses are neglected.

3) Mixing is initiated in a constant area duct at the location where the primary flow is fully expanded (primary flow pressure is equal to the local secondary flow pressure).

4) Complete mixing occurs in a constant cross-sectional channel.

5) Ejector surfaces are adiabatic.

A more general treatment which includes the effects of skin friction, blockage, and incomplete mixing is presented in Ref. 5.

The analysis is carried out using the notation of Fig. 1, and by consideration of the laws of mass flow and energy conservation and the momentum theorem written for the mixing process in a duct of constant cross section. Mixing is assumed to be initiated at station 1, where the primary flow is properly expanded so that the primary flow pressure (P_{p1}) is equal to the local pressure of the secondary flow (P_1).

Constant Area Mixing

Assuming a uniform distribution of flow properties for each initial flow prior to mixing and for the completely mixed flow at station 2, the conservation of mass flow can be expressed as

$$\dot{m}_i + \dot{m}_p = \rho_1 X_1 U_1 + \rho_{p1} a_1 V_{p1} = \rho_2 X_2 U_2 \tag{1}$$

where

$$X_2 = X_1 + a_1 = \alpha a_1 = \alpha_* a_* = \alpha_\infty a_\infty \tag{2}$$

by virtue of the assumed constant cross section.

Under the assumptions of adiabatic ejector surfaces and calorically perfect gases, the law of energy conservation can be written as

$$\dot{m}_p T_{0p} + \dot{m}_i T_{01} = (\dot{m}_p + \dot{m}_i) T_{02} \tag{3}$$

and assuming the primary flow at station 1 to be fully expanded (pressure is constant across station 1), and neglecting skin friction, the momentum theorem can be expressed as

$$(P_1 - P_2) X_2 + \dot{m}_i U_1 + \dot{m}_p V_{p1} - (\dot{m}_i + \dot{m}_p) U_2 = 0 \tag{4}$$

Since the gases are assumed to be thermally perfect, using the equation of state and the continuity equation, Eq. (1),

$$P_1 = P_{p1} = \rho_{p1} R T_{p1} = \dot{m}_p R T_{p1} / V_{p1} a_1 \tag{5}$$

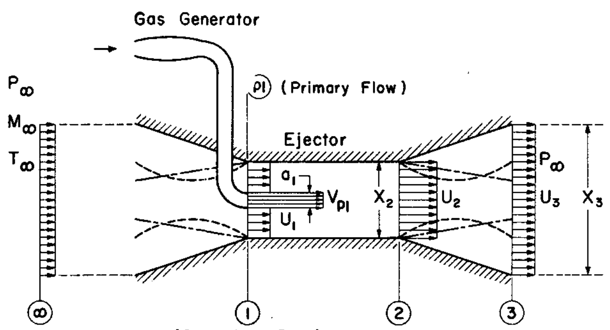

Fig. 1 Schematic representation of ejector.

and

$$P_2 = \rho_2 R T_2 = (\dot{m}_i + \dot{m}_p) R T_2 / U_2 X_2 \tag{6}$$

and R is the gas constant ($= C_p - C_v$) in appropriate units.

Expressing the velocities in terms of Mach numbers and temperatures,

$$V \text{ or } U = M\sqrt{\gamma R T} \tag{7}$$

Further, expressing the temperatures in terms of the stagnation temperature and Mach number, the Mach number M_2 at the conclusion of mixing can be expressed in terms of the conditions at station 1 by rearrangement of Eq. (4). The resulting expression is

$$A(\gamma M_2^2)^2 + B(\gamma M_2^2) + 1 = 0 \tag{8}$$

where

$$A = 1 - J^2(\gamma - 1)/2 \quad \text{and} \quad B = 2 - \gamma J^2$$

$$J = \frac{1}{1+r}\sqrt{\frac{T_{0p}}{T_{02}}}\left[\sqrt{\frac{T_{01}}{T_{0p}}}\frac{rM_1}{\sqrt{1+\frac{\gamma-1}{2}M_1^2}} + \frac{M_{p1}}{\sqrt{1+\frac{\gamma-1}{2}M_{p1}^2}}\left(1+\frac{\alpha}{\gamma M_{p1}^2}\right)\right] \tag{9}$$

$$r = (\alpha - 1)\left(\frac{M_1}{M_{p1}}\right)\sqrt{\left(\frac{T_{0p}}{T_{01}}\right)\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_{p1}^2}} \tag{10}$$

and

$$\frac{T_{02}}{T_{0p}} = \frac{1 + r\left(\frac{T_{01}}{T_{0p}}\right)}{1+r} \tag{11}$$

If conditions at station 1 are specified, the Mach number (M_2) at station 2 can be expressed as the solution of the quadratic Eq. (8) in the form

$$M_2 = \sqrt{\frac{-B \pm \sqrt{B^2 - 4A}}{2\gamma A}} \tag{12}$$

Thus it can be observed that for any given set of flow properties (M_1, M_{p1}, T_{0p}, T_{01}) and area ratio (α), there are two possible Mach numbers (M_2) after complete mixing. These Mach numbers will be referred to as $M_{2(\mathrm{I})}$ when the negative sign in Eq. (12) is used and $M_{2(\mathrm{II})}$ when the positive sign in Eq. (12) is used.

Obviously, M_2 has no real value when $B^2 < 4A$ and has two real solutions when $B^2 > 4A$. These two real solutions represent flows at station 2, which in one case is supersonic and in the other is subsonic, as can be shown by their relationship

$$M_{2(\mathrm{I})}^2 = \frac{(\gamma - 1) M_{2(\mathrm{II})}^2 + 2}{2\gamma M_{2(\mathrm{II})}^2 - (\gamma - 1)} \tag{13}$$

which is the relationship between Mach numbers across a normal shock wave. However, when $B^2 = 4A$, M_2 has only one real value, and it can be shown that at this condition, $M_{2(\mathrm{I})} = M_{2(\mathrm{II})} = 1$, since

$$J = J_c = (1/\gamma)\sqrt{2(\gamma + 1)} \quad \text{when} \quad B^2 = 4A \tag{14}$$

Thus, it is evident that in a constant area mixing duct, the mixed flow will choke ($M_2 = 1$); when $B^2 = 4A$ or $J = J_c$.

The solution representing a subsonic mixed flow (subscript I) always satisfies the second law of thermodynamics, and is referred to as the first solution, due to its widely recognized application. The solution representing a supersonic mixed flow (subscript II) is referred to as the second solution, and satisfies the second law of thermodynamics only under certain inlet conditions, as will be discussed in later sections of this document.

Having the value of the Mach number (M_2) at the conclusion of the mixing process, the pressure ratio can be evaluated using Eqs. (5) and (6) as follows.

$$\frac{P_2}{P_1} = \frac{(1+r)M_{p1}}{\alpha M_2}\sqrt{\left(\frac{T_{02}}{T_{0p}}\right)\frac{1+\dfrac{\gamma-1}{2}M_{p1}^2}{1+\dfrac{\gamma-1}{2}M_2^2}} \tag{15}$$

where T_{02}/T_{0p} is given by Eq. (11), and r by Eq. (10).

The temperature at station 2 may be evaluated in terms of the temperature at station 1, as follows.

$$\frac{T_2}{T_1} = \frac{1+\dfrac{\gamma-1}{2}M_1^2}{1+\dfrac{\gamma-1}{2}M_2^2}\left(\frac{T_{02}}{T_{01}}\right) \tag{16}$$

and

$$\frac{T_2}{T_{p1}} = \frac{1+\dfrac{\gamma-1}{2}M_{p1}^2}{1+\dfrac{\gamma-1}{2}M_2^2}\left(\frac{T_{02}}{T_{0p}}\right) \tag{17}$$

Depending upon the initial properties of the unmixed flows at station 1, either or both solutions [Eq. (12)] may represent realistically achievable flows or, in some cases, may represent a state which is not realistically achievable from the given initial conditions. To determine the validity of solutions, it is essential to investigate the entropy production in the light of the second law of thermodynamics.

It can be shown that the change of total entropy due to the mixing process is

$$\frac{\Delta S}{\dot{m}_p R} = \left(\frac{\gamma}{\gamma-1}\right)\ln\left(\frac{T_2}{T_{p1}}\right) + \left(\frac{\gamma r}{\gamma-1}\right)\ln\left(\frac{T_2}{T_1}\right) - (1+r)\ln\left(\frac{P_2}{P_1}\right) \tag{18}$$

Only those flows in which $\Delta S \geq 0$ are considered realistic. Other flows where $\Delta S < 0$ are discarded as being in violation of the second law of thermodynamics.

The equations presented thus far provide a means for evaluation of the flow properties after complete mixing in a constant area duct from known flow properties in this duct at the start of mixing. It is necessary now to describe methods for relating the initial flows to their source and for efficient discharge of the mixed flow to ambient pressure.

Isentropic Inlet

As will be described in a later section of this document, maximum ejector performance under any given set of freestream and injected primary gas characteristics requires specific conditions for the secondary flow at the start of mixing. It is the function of the inlet of the ejector to ingest fluid from the environment and to accelerate or decelerate this ingested mass flow to the required condition. In accomplishing this process, the ingested fluid will encounter a loss of momentum as a result of skin friction, blockage, or wave losses. These losses can be taken into consideration by the use of experimentally evaluated empirical factors; however, for the purposes of this presentation, they will be neglected in the interest of simplicity and to permit the development of an understanding of the fundamentals of optimization of ejector design.

Under the assumption of an adiabatic inlet, the stagnation temperature of the secondary flow at station 1 (T_{01}) is equal to the freestream stagnation temperature ($T_{0\infty}$). Ideally, for an isentropic secondary flow, the stagnation pressure at station 1 (P_{01}) is equal to the freestream stagnation pressure ($P_{0\infty}$) and, therefore, if M_1 is the desired Mach number at station 1, then

$$\frac{P_1}{P_\infty} = \left[\frac{1+\dfrac{\gamma-1}{2}M_\infty^2}{1+\dfrac{\gamma-1}{2}M_1^2}\right]^{\gamma/(\gamma-1)} \tag{19}$$

which expresses P_1 in terms of M_1.

Under the same assumptions M_{p1} can be expressed in terms of M_1. Since P_1 is equal to P_{p1},

$$M_{p1} = \sqrt{\frac{2}{\gamma-1}\left[\left(\frac{P_{0p}}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]} \tag{20}$$

These expressions relate the flow characteristics at station 1 to the given properties of the freestream or flight conditions and the characteristics of the primary, energized discharge from the gas generator. Obviously the relationship between M_1 and M_∞ will determine the geometric requirements of the inlet to the ejector. Transition from subsonic to supersonic or vice versa will require a converging/diverging geometry, and a sonic throat. Otherwise the inlet may be only converging or only diverging as illustrated on Fig. 1. One further consideration is important to the design of the inlet section of a thrust augmenting ejector. This involves the desirability for relating the ejector cross section to that of the gas generator. Since the discharge from the gas generator has known characteristics, including its mass flow ($\dot{m}_p$), its stagnation temperature (T_{0p}), and its stagnation pressure (P_{0p}), and since it is now fully expanded into a pressure (P_1) different from its normal discharge pressure (P_∞), it is essential to provide a primary nozzle discharge area (a_1) which avoids any alteration of the mass flow from the gas generator.

To maintain a fixed, isentropic primary jet mass flow,

$$\frac{\alpha}{\alpha_\infty} = \frac{a_\infty}{a_1} = \frac{\rho_{p1}V_{p1}}{\rho_{p\infty}V_{p\infty}} = \frac{M_{p1}}{M_{p\infty}}\left(\frac{P_1}{P_\infty}\right)^{\frac{\gamma+1}{2\gamma}} \tag{21}$$

When the primary jet is operating at subcritical pressure ratios (when $P_{0p}/P_\infty < [(\gamma+1)/2]^{\gamma/(\gamma-1)}$) the quantity a_* is defined as the jet area when the primary flow is expanded to ambient pressure. Thus, under that condition $\alpha_* = X_2/a_* = \alpha_\infty$.

When the primary jet is operating at supercritical pressure ratios (when $P_{0p}/P_\infty > [(\gamma+1)/2]^{\gamma/(\gamma-1)}$), α_* and α_∞ are related as follows.

$$\frac{\alpha_\infty}{\alpha_*} = M_{p\infty}\left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}\left(\frac{P_\infty}{P_{0p}}\right)^{\frac{\gamma+1}{2\gamma}} \tag{22}$$

and, therefore, for supercritical pressure ratios

$$\frac{\alpha}{\alpha_*} = M_{p1}\left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}\left(\frac{P_1}{P_{0p}}\right)^{\frac{\gamma+1}{2\gamma}} \tag{23}$$

It is of interest to note that primary nozzles injecting fluid into an ejector may not require converging/diverging shapes even when the nozzle pressure ratio is supercritical. The flow entering the ejector from a converging nozzle may expand essentially isentropically to the pressure (P_1) at which the primary and secondary flows will achieve equilibrium ($P_1 = P_{p1}$). It is this region where α ($=X_2/a_1$) is determined.

Mixing prior to the region where $P_1 = P_{p1}$ is neglected for purposes of this analysis.

Isentropic Outlet

After mixing is completed and the flow properties determined, it is essential to return the mixed flow to ambient pressure efficiently. To determine the required geometry of the outlet it is necessary to evaluate the required pressure ratio for return to ambient pressure at the outlet. This may be done by the use of Eqs. (15) and (19) for the static pressure ratio and as follows for the stagnation pressure ratio.

$$\frac{P_{02}}{P_\infty} = \left(1 + \frac{\gamma - 1}{2} M_2^2\right)^{\frac{\gamma}{\gamma-1}} \left(\frac{P_2}{P_\infty}\right) \tag{24}$$

Neglecting losses and assuming isentropic discharge from station 2 to station 3 where the pressure is P_∞, the exit Mach number is

$$M_3 = \sqrt{\frac{2}{\gamma - 1}\left[\left(\frac{P_{02}}{P_\infty}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]} \tag{25}$$

If the outlet flow is isentropic the required area ratio is determined with the use of the continuity equation as follows,

$$\frac{X_3}{X_2} = \frac{M_2}{M_3}\left(\frac{P_2}{P_\infty}\right)^{\frac{\gamma+1}{2\gamma}} \tag{26}$$

Thus, the geometric requirements of the outlet can be evaluated from a knowledge of the flow characteristics (M_2 and P_2/P_∞) at the conclusion of the mixing process under the assumption that the return of the mixed flow to ambient pressure is isentropic. The exit Mach number can also be evaluated from a knowledge of the stagnation pressure ratio as indicated by Eqs. (24) and (25). It should be noted that in an ideal flow a sonic throat is required when the mixed flow at station 2 is supersonic and the discharged flow at station 3 is subsonic or vice versa.

Ejector Performance

The performance of a thrust augmenting ejector is evaluated by comparison of its total momentum increment to the momentum increment of its gas generator when discharging to ambient pressure. This ratio, called thrust augmentation (ϕ), can be expressed as

$$\phi = \frac{(\dot{m}_p + \dot{m}_i)(U_3 - U_\infty)}{\dot{m}_p (V_{p\infty} - U_\infty)}$$

$$= \left\{(1 + r)\left[M_3\sqrt{\frac{T_{02}}{T_\infty}}\left(\frac{P_\infty}{P_{02}}\right)^{\frac{\gamma-1}{\gamma}} - M_\infty\right]\right\}$$

$$\div \left[M_{p\infty}\sqrt{\frac{T_{0p}}{T_\infty}}\left(\frac{P_\infty}{P_{0p}}\right)^{\frac{\gamma-1}{\gamma}} - M_\infty\right] \tag{27}$$

for an "air breathing" gas generator, when both the ejector and its gas generator are operating at the same freestream conditions, utilizing the same energized mass flow at the same stagnation conditions.

If the primary jet of the ejector is non air-breathing (rocket) the expressions for thrust augmentation must be modified by the elimination of the so-called "ram" drag terms associated with the mass flow of the ejector's gas generator.

Optimization of Thrust Augmentation

To evaluate the maximum performance at any given flight and injected gas conditions it is desirable, at first, to consider the ideal, lossless situation and to investigate the influence of ejector geometry on thrust augmentation. Later, the influence of losses of momentum due to skin friction, blockage, and shock waves (if they exist) on ejector performance and optimal geometric requirements will be discussed.

Under the assumptions that the flight and injected gas conditions and α_* are specified, there exists only one free parameter for the determination of a unique solution to Eq. (12). Using M_1 (the Mach number of the secondary flow at the start of mixing) as that parameter, ejector thrust augmentation is evaluated as a function of M_1, to determine whether a maximum or limit exists.

Figure 2 illustrates typical, ideal thrust augmentation and flow characteristics resulting from the injection of low temperature gas into an ejector, as a function of the Mach number of the secondary flow at the start of mixing. Under the first solution the flow exists only between the limits of $0 \leq M_1 \leq 1.520$ for this example. Values of M_1 are limited at the upper range by the physical restraint that M_3 must be greater than zero. Values of M_1 larger than 1.520 result in a deficiency of kinetic energy for traversing the adverse pressure ratio encountered at the outlet. As illustrated on Fig. 2, this first solution produces a maximum thrust augmentation at a subsonic value of M_1 ($=0.9801$), with rapid deterioration resulting from a departure from the optimal value of M_1. The Mach number (M_2) at the conclusion of mixing is always subsonic and the entropy change resulting from mixing is always positive for flows derived under the first solution. The variation of M_1 with fixed flight and injected gas characteristics results in variations of the ejector geometry as well as the performance, as illustrated on Fig. 2.

For example, at the optimum point ($M_1 < 1$, $P_1 < P_\infty$, and $M_\infty = 0$) the inlet is converging, subsonic, and accelerating. Since $M_2 < 1$, $M_3 < 1$, and $P_2 < P_\infty$, the outlet is a divergent subsonic diffuser. Variations of M_1 from the optimal value obviously will change the pressures and Mach numbers, and, therefore, the inlet and outlet shapes.

Under the second solution, a local maximum thrust augmentation exists at a transonic or supersonic value of M_1 and a limiting thrust augmentation (limited by the second law of thermodynamics) exists at a subsonic value of M_1, as shown typically on Fig. 2. In this particular example the "local" maximum performance occurs at $M_1 = 1.105$ with

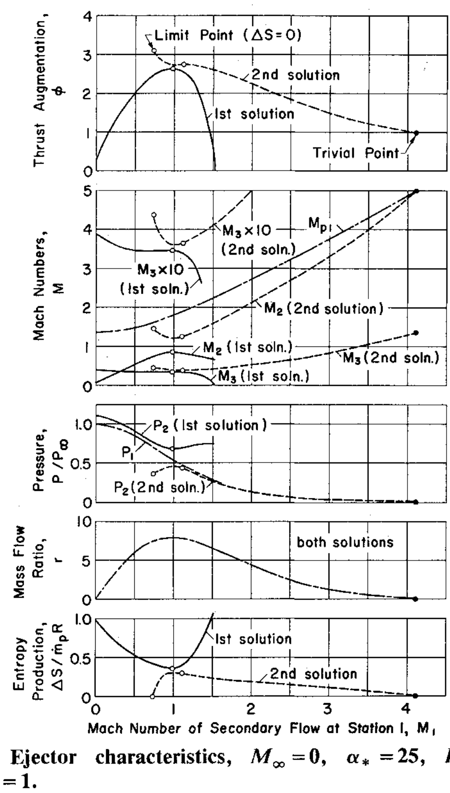

Fig. 2 Ejector characteristics, $M_\infty = 0$, $\alpha_* = 25$, $P_{0p}/P_\infty = 3$, $T_{0p}/T_\infty = 1$.

$M_2>1$ and $M_3<1$. Thus, the ejector must have a convergent/divergent inlet and outlet. The limit point ($\Delta S=0$) occurs at $M_I=0.7402$, with $M_2>1$, $M_3<1$; and since $p_I<P_\infty$, the inlet is convergent, subsonic, and accelerating, but the outlet is a convergent/divergent supersonic diffuser.

Figure 3 illustrates typical, ideal thrust augmentation and flow characteristics resulting from the injection of heated gas into an ejector, as a function of the Mach number (M_I) of the secondary flow at the start of mixing. The injection of heated primary gas results in the possibility of choking of the mixed flow in a region near $M_I=1$. As shown on Fig. 3, for example, there exists a range of values of M_I ($0.8129<M_I<1.195$) in which mixing can not proceed to its conclusion due to the choking phenomenon. The limit points of this region are specified by Eq. (14). The flow properties at the boundaries of the choked region are identical for ejector configurations derived under either the first or second solutions since $B^2=4A$, and only one solution to Eq. (8) exists.

In this particular example, the first solution is comprised of two segments. One segment ($0\leq M_I\leq 0.8129$) containing the lower choking point and the other segment ($1.195\leq M_I\leq 1.467$) containing the upper choking point, and restrained at its largest value of M_I by the requirement that $M_3\geq 0$. Each of the segments contains a local maximum performance point. However, the maximum performance point on the upper segment is at the choking point where $M_I=1.195$, which is identical to the point of optimal performance for the second solution, since both solutions coincide at this choking point. Therefore, this local optimum will be treated as a local optimum performance point for the second solution. The optimum performance of the first solution is considered as being in the segment which contains the lower choking point. A convention which will be adhered to in this document when a choking region exists. Thus, in accordance with this convention, the optimum performance of the first solution occurs at $M_I=0.7859$, for flows corresponding to the type illustrated on Fig. 3. Incidentally, the local optimum defined in this manner also represents the absolute optimal performance for first solution ejectors. In this example the inlet is accelerating, subsonic (converging) and the outlet is a diverging, subsonic diffuser.

The second solution displays three segments for the conditions illustrated on Fig. 3. The lowest segment ($0.5289\leq M_I\leq 0.8129$) contains the lower choking point and the limit point ($\Delta S=0$), while the upper choking point is contained in the segment where ($1.195\leq M_I\leq 1.920$). The upper restriction of this segment results from the fact that the flow has insufficient kinetic energy to discharge at ambient pressure in the range ($1.920<M_I<3.710$), since $M_3<0$ in that region. The third segment ($3.710\leq M_I\leq 4.113$) is restricted at its lower end by the deficiency of kinetic energy ($M_3<0$) for return of the mixed flow to ambient pressure, and at its upper end by the "trivial" point which reflects the fact that the mixing duct is completely filled by the expanded primary fluid in that duct and $\phi=\alpha=1$ and $\Delta S=r=0$. This segment is unimportant to the thrust augmenting ejector since the thrust augmentation is very close to 1 and will be neglected in the present analysis.

The local optimal performance of the segment containing the upper choking point ($1.195\leq M_I\leq 1.920$) occurs at the upper choking point ($M_I=1.195$) and requires a converging/diverging, accelerating inlet and a diverging, subsonic diffusing outlet since $M_2=1$. The subsonic segment of the second solution lies in the range ($0.5289\leq M_I\leq 0.8129$), and has a maximum performance at the limit point ($\Delta S=0$), where $M_I=0.5289$, and requires a converging, subsonic accelerating inlet and a converging/diverging, supersonic diffuser as an outlet.

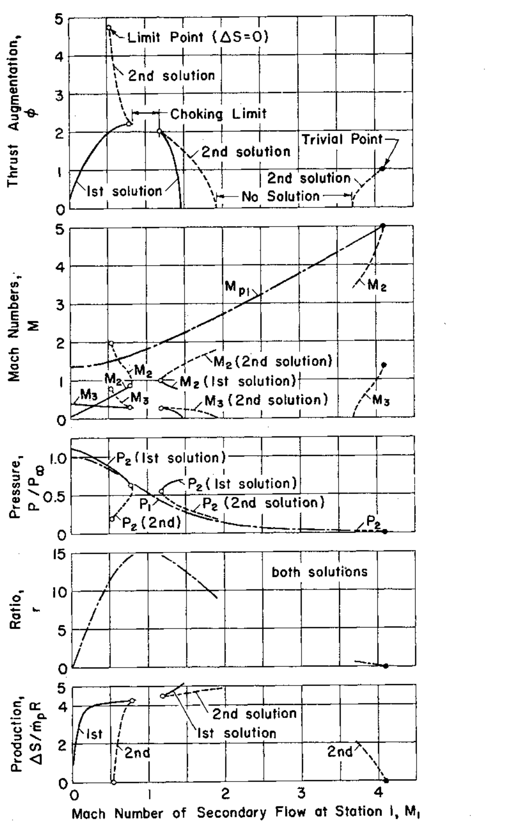

Fig. 3 Ejector characteristics, $M_\infty=0$, $\alpha_*=25$, $P_{0p}/P_\infty=3$, $T_{0p}/T_\infty=3.7$.

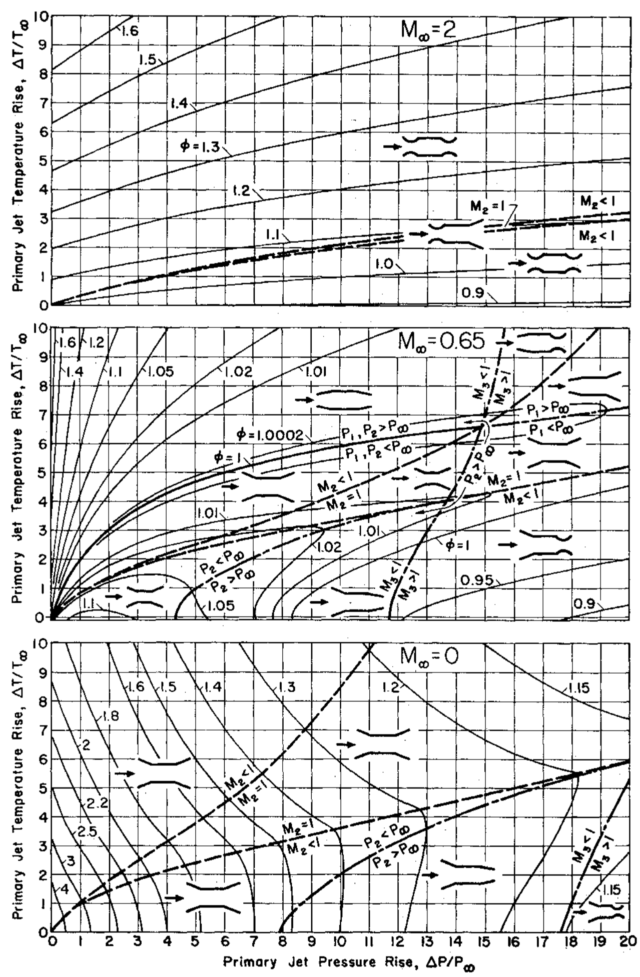

Fig. 4 Ideal thrust augmentation, optimal first solution ejectors, $\alpha_*=25$.

In summary, for a given mixing duct size relative to the primary jet size (α_* or α_∞), realistic solutions to the ejector flow problem exist within a limited range or ranges of values of M_I (or corresponding inlet and outlet configurations), as a result of physical constraints. These constraints are as follows.

1) The second law of thermodynamics must not be violated ($\Delta S \geq 0$).

2) The effluent flow must return to ambient pressure ($M_3 \geq 0$ or $P_{02} \geq P_\infty$).

3) The flow cannot exist within the choked region ($J < J_c$).

4) Limit is imposed by the trivial point ($\phi = \alpha = 1$, $\Delta S = r = 0$).

Although the trivial point appears mathematically (on Figs. 2 and 3) to be identical to the limit point imposed by the second law of thermodynamics ($\Delta S = 0$), its physical significance obviously is different. The trivial point can exist on both the first and second solution of the ejector flow problem. Figures 2 and 3 illustrate the situation where the trivial point occurs at a supersonic value of M_I, under the second solution. An example illustrating a trivial point which occurs under the first solution will be shown on Fig. 7 at a low subsonic value of M_I.

Each of the preceding constraints limit ejector performance to a range of values of M_I bounded on each end by a constraint. Analysis have indicated that within realistic ranges of M_I there are three distinct points (or ejector configurations) which can characterize the optimal thrust augmentation. These special points are defined as follows.

1) Optimal performance under the first solution: This condition always occurs at subsonic values of M_I as illustrated by example on Figs. 2 and 3. The optimal point can also occur at the lower choking point when choking exists.

2) Optimal performance under the second solution with transonic or supersonic secondary flow at the start of mixing: This point represents a local maximum performance when choking is absent, as illustrated on Fig. 2. In this case the point may be transonic or supersonic (roughly $M_I > 0.9$ for the range of data presented in this document). When the mixed flow exhibits a choking region, the second solution is divided into at least two regions as a function of M_I. One region contains the lower choking point and the other contains the upper choking point. The region which contains the upper choking point may be further divided into two segments as a result of the constraint that $M_3 \geq 0$ or $P_{02} \geq P_\infty$, as shown on Fig. 3. In the situation where the upper region is divided into two segments, one segment contains the upper choking point and the other the trivial point. In the present analyses the segment containing the trivial point is disregarded and the optimization process considers only the segment containing the upper choking point. Within the conditions considered in this document, the local optimum performance point always occurs at the upper choking point, which may be transonic or supersonic. The so-called "optimum" performance point defined in this manner is not always "productive" (thrust augmentation can be less than 1), however, it will be discussed in order to provide continuity to the optimization process. Further, the segment of the solution containing the upper choking point may not exist due to the requirement that $M_3 \geq 0$, even though the segment containing the trivial point does exist. In this case it is considered as having "no solution" at this optimum point. Another significant feature of this optimum point is that the performance at this point, when it is subsonic, is virtually equal to that at the sonic point ($M_I = 1$). Therefore, this point can be considered as an optimum point with supersonic M_I, thus simplifying the inlet design when supersonic flight is considered.

3) Limiting performance of the second solution: This particular point is established by the second law of thermodynamics for the nontrivial solution and to our knowledge always occurs at a subsonic value of M_I, as illustrated on Figs. 2 and 3.

It is important to note that each of the three special points represent configurations which have application to minimal loss ejectors depending upon the flight regime and the injected gas characteristics to be encountered during operation. For example, during subsonic flight, it may be desirable to utilize subsonic secondary flow at the start of mixing to avoid the requirement for a converging/diverging inlet. For the same reason, the use of supersonic secondary flow at the start of mixing may be desirable during supersonic flight. In this manner, the requirement for a sonic throat may be avoided by proper selection of the ejector criterion. Outlet configurations also may be less complex if second solution criteria are utilized when supersonic exit flow is required for return of the mixed flow to ambient pressure.

First Solution Thrust Augmenting Ejector Performance

Consideration of the ideal performance of thrust augmenting ejectors involves a large number of independent parameters. The number of these parameters can be considerably reduced by assuming the inlet and outlet geometry (or M_I) to be specified by the optimal or limiting design criteria previously discussed. Thus, there remain only the flight and injected gas characteristics and the ejector cross section as independent variables. Using this technique, lines of constant thrust augmentation were mapped for various primary jet stagnation properties and flight Mach numbers for ejectors with a fixed value of α_* ($=25$).

It is important to note that the maps presented in this document refer to ejectors whose cross section is fixed with respect to a_*, where the ratio of the ejectors mixing duct cross section (X_2) is only 25 times that of the cross section (a_*) of the injected flow. At high primary nozzle pressure ratios this restriction results in a large increase of the primary fluid cross section within the mixing duct and a correspondingly small secondary flow. In other words, the quantity α ($=X_2/a_I$) approaches 1.0 at large primary flow pressure ratios and, thus, the thrust augmentation decreases to values approaching 1.0 at these pressure ratios. Using larger values of α_* would result in improvement of the ideal thrust augmentation at high primary nozzle pressure ratios.

As previously indicated, the first solution, which always results in a sonic or subsonic flow after complete mixing, has one optimal performance point that always occurs at subsonic values of the secondary flow at the start of mixing ($M_I < 1.0$). Maps illustrating the ideal thrust augmentation for first solution ejectors having $\alpha_* = 25$, and translating at flight Mach numbers of 0, 0.65, and 2 are presented on Fig. 4.

Ejectors at rest with respect to the undisturbed medium or ejectors whose thrust vector is normal to the flight direction are included in the category of "stationary" ejectors ($M_\infty = 0$), since both conditions avoid the influence of the so-called "ram" drag effect on their thrust vector. As indicated on Fig. 4, they produce very acceptable performance over the entire practical range of injected gas characteristics. Under the first solution the ideal thrust augmentation achievable by stationary ejectors is a maximum in the region of low stagnation pressure and temperature and decreases with increasing stagnation pressure over the entire range studied. At high stagnation pressures and moderate stagnation temperatures, however, the ideal thrust augmentation increases with increasing stagnation temperature, as illustrated on Fig. 4. It is also of interest to note that the optimal performance of stationary ejectors occurs at the lower choking point ($M_2 = 1$) in the center region of the map. Above and below that region the optimal performance occurs when the mixed flow is subsonic.

The inlets of stationary ejectors are always accelerating and (under the first solution) the ideal outlets are decelerating over most of the range of injected gas characteristics. However, at high primary jet pressure ratios and low primary jet temperature ratios the ideal outlet becomes converging or converging/diverging.

Thrust augmenting ejectors designed under the first solution criterion encounter their most difficult operational conditions in the midsubsonic range of flight speeds. At these translational velocities, the beneficial effects of ram compression tend to be counterbalanced by their adverse effects. It is important to note, however, that ejectors designed under the first solution encounter a radical change of the influence of primary jet stagnation temperature on performance. Increases of primary jet stagnation temperature can result in improved performance, in certain regions of the pressure-temperature map.

Figure 4 also presents a map of iso-augmentation lines on a surface of primary jet stagnation temperature ratio (from $\Delta T/T_\infty = -0.0845$ to 10, or $T_{0p}/T_\infty = 1$ to 11.0845) and primary jet pressure ratio (from $\Delta P/P_\infty = 0$ to 20, or $P_{0p}/P_\infty = 1.3283$ to 21.3283), for ejectors translating at a Mach number of 0.65 with respect to the undisturbed flow. In that map the ejector performance is characterized by a curve where $P_l = P_\infty$. In the lower portion of the map it appears similar to that of the stationary ejector, while in the upper portion of the map the ideal, optimal thrust augmentation increases with increasing primary jet stagnation temperature. Thus, the region in which performance improves with increasing primary jet stagnation temperature becomes important as the translational velocity increases. The geometry of ideal ejectors translating at a Mach number of 0.65 with respect to the undisturbed medium is also depicted schematically on Fig. 4. As illustrated in the lower region of the map the required geometry is similar to that of the stationary ejector since it includes accelerating inlets. In the upper region of the map where stagnation temperatures are somewhat higher, the flow requires a compressive inlet.

At supersonic translational velocities the beneficial effects of ram compression and of increasing primary jet stagnation temperature are evident over the entire range of the pressure-temperature map, as is illustrated for optimal first solution ejectors on Fig. 4. These optimally configured ejectors demonstrate very acceptable performance at supersonic flight Mach numbers provided the primary jet stagnation temperature is sufficiently large and the geometry is optimized. At supersonic flight speeds, with subsonic flows before and after mixing, the ideal inlets are supersonic diffusers and the outlets are supersonic nozzles. Both must be of the converging/diverging type with a sonic throat, over virtually the entire range of injected gas characteristics, as indicated schematically on Fig. 4.

Effect of Translational Motion

Translational motion in the thrust direction has a considerable effect on thrust augmenting ejector performance and configuration as shown on Fig. 4. This effect on ejector performance and optimal configuration can be shown by mapping lines of constant thrust augmentation on a surface of freestream Mach number and primary nozzle pressure ratio, with fixed values of ejector area ratio (α_*) and primary jet stagnation temperature ratio (T_{0p}/T_∞). This type of map is presented on Figs. 5 and 6 for primary jet conditions representing cold laboratory air ($T_{0p} = T_\infty$), and heated air ($T_{0p}/T_\infty = 3.7$), respectively. The condition $T_{0p} = T_\infty$ represents conditions in which the primary air is supplied from a storage vessel at ambient temperature. The heated air temperature ratio is chosen to represent a primary jet stagnation temperature of 816°C (1500°F) at sea level, and correspondingly much lower temperatures at high altitude; temperatures consistent with those easily achievable under laboratory conditions.

Cold Primary Gas Ejectors

As can be observed on Fig. 5, the highest ideal thrust augmentation achievable by first solution ejectors with cold gas injection is achieved at the lowest freestream Mach numbers and low nozzle pressure ratios. Since the primary stagnation temperature is held fixed at a value equal to the ambient temperature (stored air), the beneficial influence of high temperatures previously described on Fig. 4 cannot be observed in this type of laboratory condition and thrust augmentation decreases monotonically with increasing freestream Mach number throughout the range considered in this map.

Ejector geometry, depicted schematically on Fig. 5, is determined by the values of the Mach numbers and pressure ratios across the inlet and outlet of the ejector. Since this map refers to the first solution only, the optimal Mach number of the secondary flow at the start of mixing and the Mach number after complete mixing are subsonic.

The inlet is accelerating at low subsonic freestream velocities and diffusing at high subsonic freestream velocities, where the secondary flow at the start of mixing must be decelerated to achieve optimal conditions. At supersonic freestream velocities, the achievement of the required subsonic secondary flow at the start of mixing requires a converging/diverging (compression inlet) if the ingestion process is to be isentropic. Similarly, since the flow after complete mixing is always subsonic under this first solution, the outlet is either converging, diverging, or converging/diverging. However, the converging/diverging outlets shown on the map are supersonic nozzles since $P_2 > P_\infty$ and $M_3 > 1$.

The assumption of this map (Fig. 5) is $T_{0p} = T_\infty$. It represents a highly cooled primary air supply at high freestream Mach numbers since

$$\frac{\Delta T}{T_\infty} = \frac{T_{0p} - T_{0\infty}}{T_\infty} = -\frac{\gamma - 1}{2} M_\infty^2 \quad \text{for} \quad T_{0p} = T_\infty \tag{28}$$

If the primary jet is to have a positive net thrust ($V_{p\infty} > U_\infty$), the primary nozzle pressure ratio must have a value sufficiently larger than that of the freestream, as indicated by the line marked "Primary Thrust = Its Ram Drag" which represents the condition $V_{p\infty} = U_\infty$.

Heated Primary Gas Ejectors

The injection of heated gas brings into focus the phenomenon of choking as illustrated on Fig. 6. As shown, there exists a region in which optimal performance occurs at the lower choking point ($M_2 = 1$). In addition, a distinct band roughly surrounding the line where $P_l = P_\infty$, in which the thrust augmentation does not exceed 1.00023, is evident on Fig. 6 under the stated conditions. Above that band the ejector requires compression inlets and the thrust augmen-

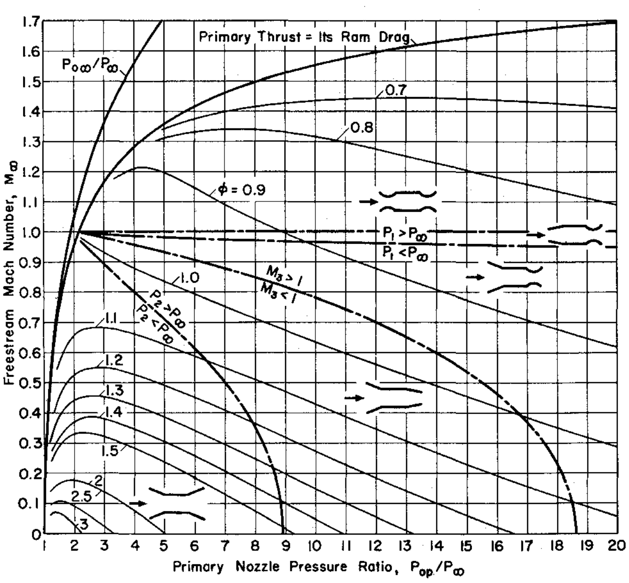

Fig. 5 Ideal thrust augmentation, optimal first solution ejectors, $\alpha_* = 25$ and $T_{0p} = T_\infty$.

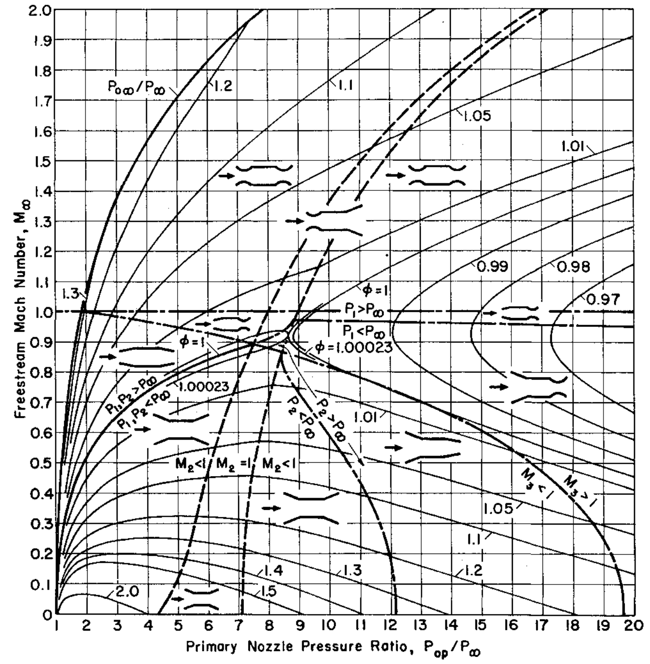

Fig. 6 Ideal thrust augmentation, optimal first solution ejectors, $\alpha_* = 25$ and $T_{0p}/T_\infty = 3.7$.

tation increases with increasing flight speeds, while below that band the ejectors operate with accelerating inlets and the ideal thrust augmentation decreases with increasing flight speeds.

In general, the ideal performance of first solution ejectors is very acceptable at low subsonic flight speeds, at the conditions stated on Fig. 6. At high subsonic and transonic flight speeds the ideal thrust augmentation of first solution ejectors is very poor and improves somewhat at supersonic speeds. It should be recognized that Fig. 6 was prepared for sea level laboratory conditions, and was not intended to be realistic for high flight Mach numbers. The temperature ratio (T_{0p}/T_∞) of the injected gas was assumed to correspond to 816°C (1500°F) at sea level, resulting in a temperature of only 528°C (982°F) at 11-km (36,000-ft) altitude. The performance at high freestream Mach numbers can be improved considerably with the use of higher primary jet stagnation temperatures as has been previously illustrated on Fig. 4.

Outlets under the first solution are required to return the subsonic or sonic mixed flow to ambient pressure isentropically. At high primary jet pressure ratios or at transonic and supersonic freestream velocities this involves a change from the subsonic flow after mixing to a supersonic flow at the exit. Thus, the outlet is obviously converging/diverging but accelerating. At low to midsubsonic freestream velocities and relatively low primary flow pressure ratios, the ideal isentropic outlet is converging (subsonic nozzle) or diverging (subsonic diffuser) as illustrated schematically on Fig. 6.

Influence of Losses on Ejector Performance

Realistic ejector performance will be somewhat degraded in comparison to that of the ideal, lossless performance previously illustrated, due to the influence of skin friction, blockage, incomplete mixing, and, where supersonic flow is involved, wave losses. Correct estimates of the realistic performance of ejectors require precise evaluation of the various loss factors which, in general, require experimental measurement. Erroneous estimates of loss factors can result in a failure to properly optimize the ejector geometry, thus resulting in large deviations from optimal performance due to the combined effects of losses and deviations from optimal geometries.

Thrust augmenting ejectors always operate at an overall pressure ratio of 1, since ingestion and discharge are always at

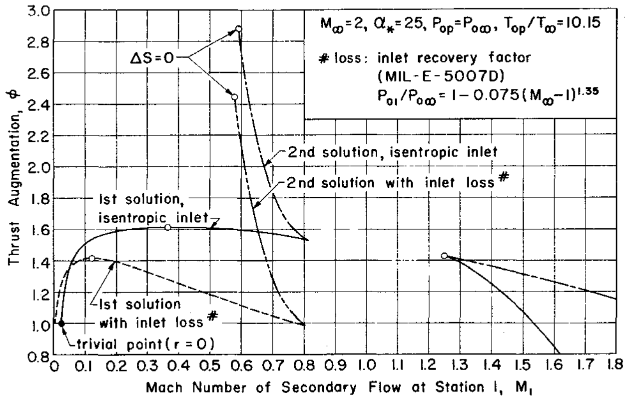

Fig. 7 Influence of inlet loss.

ambient pressure. Constant pressure throughout the cycle always results in very poor performance and, therefore, it is essential, for achievement of high performance, to achieve a high degree of compression and expansion during the cycle. The compressive regions of the ejector represent the most critical elements that may be responsible for the major losses, unless their design is carefully carried out.

In a conventional first solution ejector designed for operation at low subsonic speeds with low primary nozzle pressure ratio, the outlet generally consists of a subsonic diffuser (Figs. 4-6, for example). At high subsonic speeds, subsonic compression inlets become important for first solution ejectors (as illustrated on Figs. 4 and 6) and for second solution ejectors, as will be discussed in Ref. 8. This inlet configuration is somewhat similar to those utilized for subsonic jet engine inlets, but details of these configurations remain to be investigated.

The jet-diffuser ejector[6] was created to overcome the subsonic compression problem involved in conventional low speed ejector outlet designs. However, other major obstacles to high performance include the primary nozzle attitude and mixing, which contribute to variations of the flow patterns. In addition, skin friction and inlet blockage contribute to losses as described in Ref. 7.

Another compressive element that can influence the performance of an ejector occurs as a result of wave losses at the inlet or outlet of the ejector. Inlet wave losses occur primarily during supersonic flight and outlet wave losses occur primarily in ejectors designed under the second solution criterion which will be discussed in Ref. 8. Other wave losses that may occur in the primary flow usually can be avoided or their effect minimized by proper design using existing technology.

In general, ejectors encounter inlet losses due to skin friction, blockage, and, at supersonic flight speeds, wave losses. The magnitude of these losses and their influence on thrust augmentation depends upon the details of a particular design, such as the arrangement and geometry of the primary jet ducting and the ejector inlet shape, and upon the flight Mach number and the desired secondary flow Mach number at the start of mixing. Analyses of all possible situations are beyond the scope of this document, but to illustrate the effect of inlet losses, one particular example for which design data is available from experience in conventional engine inlet design will be presented.

Ejectors translating at supersonic speeds generally have an expansion outlet for both the first and second solutions, or at low supersonic speeds, a very weak supersonic compression outlet for second solution ejectors. Therefore, the major loss in an ejector translating at supersonic speeds occurs at its inlet; particularly if the design is based upon a criterion with subsonic secondary flow at the start of mixing.

Figure 7 illustrates the influence of inlet losses on first and second solution ejectors. The losses were evaluated with the aid of standard engine inlet compression loss specifications as required by MIL-E-5007D for inlets translating at Mach numbers between 1 and 5. The gas generator in this example is a ramjet with a stagnation temperature corresponding to 1927°C (3500°F) at an altitude of 11 km (36,000 ft).

As indicated, the inlet loss results in a modification of the optimal value of M_l and in a degradation of performance compared to the ideal case. Thus, the geometry of the inlet and outlet and the resulting performance are shown to be influenced by the inlet loss. In this particular case the performance degradation for the optimal first solution is about 12% and for the limiting second solution ejector is about 15% of the ideal performance. The thrust augmentation after accounting for the loss is still very appreciable. The modification of M_l (which is related directly to the mass flow rate of the secondary fluid and which describes the geometric specifications for the inlet and outlet configuration) is considerable for the optimal first solution ejector, while the value of M_l under the limiting second solution criterion is only slightly affected by the losses.

Discussion

This document is Part I of a presentation of the results of an investigation of the performance of thrust augmenting ejectors operating in and with compressible gases and utilizing energy from the discharge of conventional gas generators. The investigation was limited to consideration of ejectors in which the mixing process was assumed to occur in a duct of constant cross section and the examples presented are limited to a particular ratio of mixing duct cross section (X_2) to primary jet throat area (a_*), $\alpha_* = X_2/a_* = 25$. Although it has been shown that there exist two solutions to the equations representing the conservation laws of mass flow and energy and the momentum theorem, this Part I document is devoted primarily to the so-called first solution in which the flow after complete mixing is subsonic. Most importantly, it has been shown that for any given set of flight and injected gas characteristics, there exists ejector inlet and outlet geometries which provide optimal performance.

It has been shown that the ideal thrust augmentation of ejectors whose configuration is optimized under the first solution is very acceptable at virtually all flight speeds, provided the injected gas properties are selected appropriately. At low subsonic and at supersonic flight speeds, the ideal performance and the performance with consideration of the major losses is very acceptable in the range of injected gas characteristics available from conventional gas generators. However, in the midsubsonic to transonic flight speed range, it is noted that acceptable performance is achievable only with the injection of high-temperature, low pressure ratio gas. These gas characteristics are available only from gas generators having low thermal efficiency at the stated flight speed, and, therefore, may represent situations of limited practical utility.

Second solution ejectors are of significance in virtually all regions of the flight and injected gas spectrum. They offer very high performance with relatively simple geometric shapes, especially at high flight speeds, and will be discussed in detail in Ref. 8.

Acknowledgments

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